Finite Math - Fall 2018
Lecture Notes - 9/18/2018

Homework

- Section 3.1 - 79, 81
- Section 3.2 - 9, 11, 21, 23, 24, 30, 33, 35, 37, 39, 43, 65, 66, 68, 70

Section 3.1 - Simple Interest

Example 1. *Suppose a brokerage firm uses the following commission schedule*

Principal	*Commission*
Under \$3,000	*\$32+1.8% of principal*
\$3,000 - \$10,000	*\$56+1% of principal*
Over \$10,000	*\$106+0.5% of principal*

An investor purchases 75 *shares of a stock at* \$37.90 *per share, keeps the stock for* 150 *days, then sells the stock for* \$41.20 *per share. What was the annual interest rate earned on the investment? (Again, assume a* 360*-day year.)*

Solution. 6.352%

Average Daily Balance. A common method for calculating interest on a credit card is to use the *average daily balance method*. As the name suggests, the average daily balance is computed, then the interest is computed on that.

Example 2. *A credit card has an annual interest rate of* 19.99% *and interest is calculated using the average daily balance method. If the starting balance of a* 30*-day billing cycle is* \$523.18 *and purchases of* \$147.98 *and* \$36.27 *are posted on days* 12 *and* 25*, respectively, and a payment of* \$200 *is credited on day* 17*, what will be the balance on the card at the start of the next billing cycle?*

Solution. *We must figure out what the balance is on each day of the month. At the end of day* 1*, the balance is* \$523.18*. The first transaction happens on day* 12*, which is a purchase of* \$147.98*, making the balance* \$671.16*. The next transaction is on day* 17*, a payment of* \$200*, making the balance* \$471.16*. The next, and final, transaction is on day* 25 *which is a purchase of* \$36.27*, making the balance* \$507.43*. It helps to make a chart of this data*

Day 1-11:	\$523.18	*(11 days)*
Day 12-16:	\$671.16	*(5 days)*
Day 17-24:	\$471.16	*(8 days)*
Day 25-30:	\$507.43	*(6 days)*

To find the average daily balance, we can take the sum of the balance at the end of each day, then divide by the number of days.

$$SUM = 11(523.18) + 5(671.16) + 8(471.16) + 6(507.43) = \$15,924.64$$

Dividing this number by 30 *gives the average daily balance*

$$ADB = \frac{SUM}{30} = \$530.82.$$

We can use the formula for interest to figure out the interest incurred (assuming 360 *days in a year),* $t = \frac{30}{360} = \frac{1}{12}$

$$I = Prt = (530.90)(0.1999)\left(\frac{1}{12}\right) = \$8.84.$$

To find the balance at the start of the next billing cycle, we add this interest to the remaining balance at the end of the last cycle:

$$\textit{New Balance} = \underbrace{\$507.43}_{\textit{Day 30 balance}} + \underbrace{\$8.84}_{\textit{Interest}} = \$516.27$$

Example 3. *A credit card has an annual interest rate of* 19.99% *and interest is calculated using the average daily balance method. If the starting balance of a* 28*-day billing cycle is* \$696.21 *and purchases of* \$25.59, \$19.95, *and* \$97.26 *are posted on days* 6, 13, *and* 25, *respectively, and a payment of* \$140 *is credited on day* 8, *what will be the balance on the card at the start of the next billing cycle?*

Solution. \$708.92

Section 3.2 - Compound and Continuous Compound Interest

Compound Interest. In the case of simple interest, the interest is computed exactly once: at the end. Typically, however, interest is usually compounded something like monthly or quarterly.

Example 4. *Suppose* \$5,000 *is invested at* 12%*, compounded quarterly. How much is the investment worth after* 1 *year?*

Solution. *We find the future value at the end of the first quarter:*

$$A_1 = \$5,000\left(1 + 0.12\left(\frac{1}{4}\right)\right) = \$5,150.$$

This amount is carried into the second quarter and interest is computed again over the quarter:

$$A_2 = \$5,150\left(1 + 0.12\left(\frac{1}{4}\right)\right) = \$5,304.50.$$

We do this twice more to find a value at the end of the fourth quarter:

$$A_3 = \$5,304.50\left(1 + 0.12\left(\frac{1}{4}\right)\right) = \$5,463.635.$$

$$A_4 = \$5,463.635\left(1 + 0.12\left(\frac{1}{4}\right)\right) = \$5,627.54.$$

If we generalize this process, we end up with the following result

Definition 1 (Compound Interest)**.**

$$A = P(1+i)^n, \text{ where } i = \frac{r}{m}$$

The variables in this equation are

- $A =$ *future value after* n *compounding periods*
- $P =$ *principal*
- $r =$ *annual nominal rate*
- $m =$ *number of compounding periods per year*
- $i =$ *rate per compounding period*
- $n =$ *total number of compounding periods*

Alternately, one can reinterpret this formula as a function of time as

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

where A, P, r, and m have the same meanings as above and t is the time in years.

Example 5. *If* $\$1,000$ *is invested at* 6% *interest compounded (a) annually, (b) semiannually, (c) quarterly, (d) monthly, what is the value of the investment after* 8 *years? Round answers to the nearest cent.*

Solution. *In this example, the quantities that will be changing are* m *and* n *(and thus also* i*). The fixed quantities are the principal* $P = \$1,000$ *and the annual nominal rate* $r = 0.06$*.*

(a) *Annually compounded means* $m = 1$. *Since we are going for* 8 *years, this means there will be* $n = 8(1) = 8$ *compounding periods. We also get* $i = \dfrac{0.06}{1} = 0.06$, *so the future value will be*

$$A = \$1,000(1 + 0.06)^8 = \$1,593.85.$$

(b) *Semiannually compounded means* $m = 2$. *Since we are going for* 8 *years, this means there will be* $n = 8(2) = 16$ *compounding periods. We also get* $i = \dfrac{0.06}{2} = 0.03$, *so the future value will be*

$$A = \$1,000(1 + 0.03)^{16} = \$1,604.71.$$

(c) *Quarterly compounded means* $m = 4$. *Since we are going for* 8 *years, this means there will be* $n = 8(4) = 32$ *compounding periods. We also get* $i = \dfrac{0.06}{4} = 0.015$, *so the future value will be*

$$A = \$1,000(1 + 0.015)^{32} = \$1,610.32.$$

(d) *Monthly compounded means* $m = 12$. *Since we are going for* 8 *years, this means there will be* $n = 8(12) = 96$ *compounding periods. We also get* $i = \dfrac{0.06}{12} = 0.005$, *so the future value will be*

$$A = \$1,000(1 + 0.005)^{96} = \$1,614.14.$$

Example 6. *If* $\$2,000$ *is invested at* 7% *compounded (a) annually, (b) quarterly, (c) monthly, what is the amount after* 5 *years? How much interest is accrued in each case? Round answers to the nearest cent.*

Solution.

(a) \$2805.10 *with* \$805.10 *in interest.*

(b) \$2829.56 *with* \$829.56 *in interest.*

(c) \$2835.25 *with* \$835.25 *in interest.*